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THE TRIP OF ASSOCIATE SECRETARY M. H. INGRAHAM IN THE INTERESTS OF THE SOCIETY

The editors of this Bulletin have invited me to make a statement about the work I am to do during the academic year 1934–1935 in helping to put into operation the proposals for stabilizing the Society's finances. I greatly appreciate this kindness.

President Coble in his letter of May 19th to the members of the American Mathematical Society discussed the financial needs of the Society and the program which the trustees and council had adopted to meet the present emergency. He also outlined the plan which involves my visiting various American colleges and universities in order to advance the interests of the Society. It is hoped that my efforts will be helpful in initiating the plan of individual contributing membership and in securing cooperation of institutions of learning in our program for maintaining channels of mathematical publication.

Since Professor Coble wrote, a careful study of the distribution of papers in our mathematical research periodicals by universities and colleges during the last seven years has been completed, and the annual amount which a university would have to pay as a sustaining member if it were to contribute to the Society $2.75 per page of publication by its faculty members has been computed. It is of course realized that this will merely serve as a rough guide for universities and colleges in determining the size of the contributions they may wish to make. Besides this, much information has been gathered concerning the publication problems in other sciences so that university authorities can be better informed as to the whole situation which it is likely they must ultimately face. In some cases the financial conditions of learned societies are such that there is little probability that any aid will be asked from the universities. In others, such as physics and biology, universities are already helping by paying part of the publication cost for the papers of their faculty members. It is expected that by presenting as much information as possible to the mathematical groups, the members of the Society will themselves come to a clearer un-

derstanding of the problems involved in our common undertaking, thus becoming better prepared to seek the cooperation of others, and that many will desire to contribute financially to support the work of the Society.

I feel very clearly the wisdom of the principle laid down by President Coble when he said, "It is clearly understood that in dealing with the universities Professor Ingraham will deal through the mathematics departments of the universities and will be guided in his work by their advice and wishes. In some localities it may be the judgment of the local members that Professor Ingraham should directly present the needs of the Society to the administrative officers of the university; in others they may prefer that he aid the local group by furnishing information and advice without his having direct contact with anyone outside the department of mathematics." From many of the places which I have already arranged to visit I have been informed that it will be expedient for me to see the president, deans, or other administrative officers involved, but this, of course, in no way needs to be considered a precedent to be followed generally, when the judgment of the local group may be otherwise. It is my hope that in each place that I visit I may have the opportunity to discuss the work of the Society with as large a group of mathematicians as possible. It might be well for me to have a chance to speak to the group as a whole and also to any individuals who wish to ask questions concerning contributing membership, gifts to the Society, or modes of obtaining cooperation.

I think it should be clear that in a few places it would be unjust to expect either the members or the institution to do anything at present, but an understanding, at this time, of the problems might lead to further cooperation when financial conditions improve. The officers of the Society feel it would be desirable if in the future those who reach a certain degree of advancement and security in the mathematical profession should as a matter of course become contributing members.

It has been impossible for me as yet to complete my itinerary, since some very important work must be completed next fall before definite appointments for the rest of the year can be made. A rough schedule is as follows:

Late September and October	New England
Early November	Eastern Canada
November and December and early January	Middle West
Late January and early February	South
Late February and March	Middle Atlantic states
Late April and May	Mountain states and Pacific coast

I shall strive by correspondence to be in touch with many mathematical groups which because of the limitations of time it will be impossible to visit. Since I shall visit only about seventy institutions of learning and since it will be impossible for me to see more than about half of the members of the Society, it is clear that the Society can carry out this program only through an adequate response to many appeals that of necessity must be written rather than verbal and general rather than personal.

I shall keep my headquarters at the University of Wisconsin where Miss Evelyn Hull will be in charge of my correspondence and of much of the detailed work connected with this plan. Many letters which I shall write will be typed in Madison and must necessarily be mailed without my signature.

The success of this whole plan does not depend upon the efforts of any one person or small group of individuals but upon the degree to which the mathematicians of America feel that the goal is essential and cooperate to attain it.

M. H. INGRAHAM,
Associate Secretary

July 17, 1934

HOPFNER ON GEODESY

Physikalische Geodäsie. By F. Hopfner. Mathematik in Monographien und Lehrbüchern. No. 15. Leipzig, Akademische Verlagsgesellschaft, 1933. ix+434 pp.

The type of textbook on geodesy most familiar to students in this country is a sort of practical manual of large-scale higher surveying. Often considerable attention is given to the details of observation. The point of view of such books is predominantly geometric; naturally there is little space for extended theoretical developments, often the proofs of formulas have to be omitted.

More theoretical is the type of book best exemplified by the classical treatises of Helmert and Clarke. In these there is more of theory and a nearer approach to mathematical rigor, also less attention to observational detail, but the points of view of the engineer in the field and of the computer or the theorist are not entirely forgotten. This comprehensiveness of outlook is one reason why these two works retain their value fifty years and more after their publication.

Hopfner's book takes us further over into the theoretical side than either Clarke or Helmert, or indeed than any book known to the reviewer calling itself a treatise on geodesy. This is said by way of information, not of blame; the author is entitled to choose his own treatment and the first eight chapters are well done within the limitations of the space available, though the question might arise as to how much of the material of these chapters is really geodesy, or to what extent it prepares the way for a study of geodesy.

Chapter I treats of the relation of line and surface integrals, Green's theorem, the Laplacian operator, and elliptic coordinates. Chapter II deals with spherical harmonics; Chapter III with Lamé's functions; and Chapter IV with attraction and potential, and also with Green's functions and Dirichlet's principle. Chapter V, a brief one, deals with the attraction and potential of a homogeneous ellipsoid. In Chapters VI, VII, and VIII, uniform rotation is introduced, and we have a discussion of the figures of equilibrium of rotating homogeneous liquids: Maclaurin's ellipsoid, Jacobi's ellipsoid with three unequal axes, Poincaré's pear-shaped figure, etc. The foot-notes give welcome references to the more recent literature. All this is interesting, but how much connection with geodesy it has is not so clear; the author does not discuss the possible cosmogonic implications, which are usually considered to constitute the most interesting applications of these theoretical developments.

Chapter IX treats Clairaut's problem, the figure of equilibrium and the form of the level surfaces for a slowly rotating mass of self-attracting liquid. The outer surface and the level surfaces within are approximately ellipsoids of revolution. Clairaut's differential equation connecting the ellipticity with the equatorial radius of any given level surface is established and integrated for several well known laws of density that make the results mathematically tractable. These laws suppose continuous variation in density from center to free surface. Seismologists look with more favor on discontinuities in density, but this physically interesting case is passed over, except for references to the literature. Radau's ingenious transformation of Clairaut's equation is given;

it shows why very different laws of density give so nearly the same result, as far as the external gravitational field is concerned, that the observed character of the latter is no test of the merit of one or another law.

These nine chapters occupy 287 of the 429 pages in the book (exclusive of preface, table of contents, and index.) So far the reviewer has found nothing to which he could reasonably object and much to commend, but the two remaining chapters are full of controversial matter. Even the reader approaching the subject for the first time might suspect something of the sort, but he would find little help in the book if he wished to learn the other side.

Chapter X contains an error that will be more readily appreciated by mathematicians than will the fine points of the controversy about the more technically geodetic matters in Chapter XI. In Section 1 of Chapter X the author "proves" the existence of a Newtonian potential function expressed as a series of spherical harmonics that is supposed to represent within a certain region the potential both inside and outside of attracting matter. The method of obtaining this series is very simple in certain cases, especially those where the matter is homogeneous. Unfortunately it will not work, as would be shown by a comparison with the known and correct result for the external potential of a homogeneous ellipsoid obtained by a different method and given by the author himself in Section 5 of the same chapter.

This error vitiates much of Chapter XI, which follows and which is by far the longest chapter in the book. The Chapter is entitled "Das Problem von Bruns." Bruns published in 1878 a very useful little work of some fifty pages called *Die Figur der Erde*; in it he emphasized the physical rather than the purely geometric aspect of the problem and reminded geodesists that their problem involved more than the form of one level surface of the earth's field of force, in short that it included the entire field of force. Bruns's work was valuable although he made one grave error of potential theory, an error that Hopfner unfortunately adopted as his own and defended in his articles published in periodicals. Hopfner has repeatedly called attention to the value of Bruns's work. But, in spite of Bruns's service to geodesy, it seems to the reviewer that it is hardly justifiable to call a certain equation for the warping of the geoid "Bruns's theorem" and a correction based on this equation "Bruns's term," as Hopfner does. Stokes in his classic article *On the variation of gravity at the surface of the earth*, (Transactions of Cambridge Philosophical Society, vol. 8, p. 72, or Mathematical and Physical Papers, vol. 2, p. 131), nearly thirty years earlier than Bruns, used the same equation and the same term, and he did not even pause in the course of his exposition to call special attention to them.

More important, however, to the reader using this book as an introduction to the subject, is the fact that the author bases much of this chapter on the erroneous theorem already mentioned and makes other statements not generally accepted. He refers to some of his numerous articles along the same lines, but does not indicate that his conclusions have been widely questioned. It would take far too much space even to summarize the controversy. All that the reviewer can do is to put the reader on his guard, and to refer him to the volumes of Gerland's *Beiträge zur Geophysik* as far back as 1929; there he will find numerous articles by Hopfner and by those who take issue with him.

W. D. LAMBERT

SHORTER NOTICES

The Nature of Mathematics. By Max Black. New York, Harcourt-Brace, 1934. ix+219 pp.

This book contains expository accounts of the three most important schools of mathematical philosophy: the logistic, the formalist, and the intuitionist. The author calls his work a critical survey, and such it is; for the text is interwoven throughout with a web of philosophical criticism, frequently subtle and penetrating, and always interesting. The technical weaknesses of the various schools of thought are, presumably, fairly well known. But the true philosopher is concerned with much broader problems: questions of form, structure, meaning, the interrelations between different philosophies, the position of mathematics in human knowledge. Although the author treats none of these questions systematically (since his book is but an introduction), nevertheless he skillfully carries out his analysis from the viewpoint which they imply, and does so in an engaging and personal style. In this type of criticism lies his real contribution.

The most extensive account is that of the logistic thesis, that is, the aggregate of theories which assert that mathematics is a branch of logic. Here we find a simplified but sufficiently technical description of the methods of Russell and Whitehead's *Principia Mathematica*, followed by an account of the various attempts to remedy its worst defects. The difficulties which seem to be inherent to the logistic scheme can be judged by the rather desperate devices with which they are met, devices which the author ironically suggests, really belong to the technique of theology. "God arrives to solve the difficulties of Berkeleyan idealism or Earl Russell in less ambitious times invokes the axiom of reducibility." On a more technical plane the author concludes that the imperfections of the logistic scheme "can be traced back to insufficiently precise technique in manipulating systems of symbols," a situation more formidable than it first appears since it reaches back into the structure of language itself. This constant preoccupation with the analysis of systems of symbols (including languages) and with the problems of correct manipulation constitutes a tenuous thread of criticism running through the book. Mathematics may in a sense be regarded as "the grammar of all symbolic systems . . . ," or again, as "the crystallized syntax of all systems of interrelated objects."

The supplementary portions of the book include many interesting topics. In connection with formalism there is a section on the development of geometry and a note on Gödel's theorem,—a remarkable result which makes it doubtful that the formalist program can be carried through. In connection with intuitionism there is a section on the opinions of Kronecker and Weierstrass, a sketch of the theory of point sets, a summary of the *polémiques sur le transfini* of the contemporary Paris school, and finally, short accounts of the intuitionist calculus of propositions and the intuitionist theory of cardinal numbers.

P. A. SMITH

Nouvelles Tables de Logarithmes à 36 Décimals. By F. J. Duarte. Paris, Gauthier-Villars, 1933. 28+128 pp.

These tables contain the common logarithms to 36 decimals of the integers from 1 to 1000 and of the prime numbers from 1 to 10,007. There is also a table giving the logarithms to 33 decimals of factorial numbers from 3050 to 10,000 at intervals of 50 and of a few larger numbers up to 1,000,000. This is supplementary to the table published by the same author in 1927 giving the logarithms to 33 decimals of factorial numbers up to 3000.

The formulas by which the results have been calculated and checked are given, together with some auxiliary tables useful in calculating logarithms. The utmost care has been taken in the printing and the author is hopeful that the book is free from errors.

W. R. LONGLEY

NOTES

An International Congress for Problems of Vector and Tensor Analysis was held at the University of Moscow, May 17–23, 1934; it was organized by Professor B. Kagan, of the University of Moscow, and Professor J. A. Schouten. Among the active foreign participants in this Congress were Enea Bortolotti and Pietro Burgatti (Italy), Elie Cartan (France), J. A. Schouten (Holland), W. Blaschke and E. Koehler (Germany), A. Duschek (Austria), V. Hlavaty (Czechoslovakia), and S. Golab, A. Wundheiler, and A. Hoborski (Poland).

At a recent meeting of the American Mathematical Society, held at Chicago, the dinner program was in honor of Thomas F. Holgate, emeritus professor of mathematics at Northwestern University, and H. E. Slaught, emeritus professor of mathematics at the University of Chicago. Each had been secretary of the Chicago section for a period of ten years and each had served his university continuously for a period of more than forty years. Addresses were made by Professors F. R. Moulton, L. E. Dickson, D. R. Curtiss, G. A. Bliss, and W. C. Graustein. Professors Holgate and Slaught responded.

The Italian Society of Sciences has awarded its prize in mathematics for 1932 to Professor G. Krall, of the University of Rome.

Professor Paul Langevin, of the Collège de France, has been elected a member of the Paris Academy of Sciences, in the section of general physics.

The University of Glasgow has conferred an honorary doctorate on Professor H. M. Macdonald, of the department of mathematics at the University of Aberdeen.

Karl Pearson, emeritus Galton professor of eugenics at the University of London, has been elected an honorary member of the Edinburgh Royal Society.

Princeton University has conferred an honorary doctorate of science on Professor A. H. Compton, of the University of Chicago.

Yale University has conferred an honorary doctorate of science on F. G. Keyes, professor of physical chemistry at the Massachusetts Institute of Technology.

Dr. Nathan Jacobson and Dr. J. B. Rosser have been awarded Proctor fellowships at Princeton University.

Dr. C. Grace Shover, of Connecticut College for Women, has been made Emmy Noether fellow in mathematics at Bryn Mawr College.

Dr. Guido Beck, professor of physics at the German University of Prague, has been appointed resident professor of mathematical physics at the University of Kansas for 1934–35.

Dr. G. Boaga, privat docent in Padua, and Dr. P. Tortorici, privat docent in Palermo, have been appointed associate professors of topography and geodesy, in the engineering schools of Pisa and Palermo, respectively.

The title of Stokes lecturer in mathematics has been conferred on Dr. Max Born by Cambridge University.

Dr. Richard Courant, formerly director of the Mathematical Institute of the University of Göttingen, has been appointed visiting professor of mathematics at New York University.

Dr. Robert Haussner, professor of mathematics at the University of Jena, has retired.

Dr. Theodor von Kármán, professor of mechanics and technical aerodynamics at the Aachen Technical School, has retired.

Associate Professor Franz Krauss has been promoted to a professorship of mathematics at the Aachen Technical School.

Professor G. Mammana, of the University of Cagliari, has been transferred to the chair of higher analysis at the University of Catania.

Professor A. Maroni, privat docent in Florence, has been appointed associate professor of analytic geometry at the University of Messina.

Dr. G. C. McVittie has been appointed lecturer in applied mathematics at the University of Liverpool.

Professor G. Mignosi, of the University of Palermo, has been promoted to a professorship of algebra.

Mr. R. O. Street, of the University of Liverpool, has been appointed to the chair of mathematics at the Royal Technical College, Glasgow.

Dr. R. A. Beaver has been promoted to an assistant professorship at the New York State College for Teachers, Albany.

Professor Gregory Breit, of New York University, has been appointed professor of physics at the University of Wisconsin.

Miss Mildred Carlen, of the department of mathematics at Brown University, has been appointed registrar of the Graduate School.

Associate Professor Evelyn Carroll-Rusk has been promoted to a professorship of mathematics at Wells College.

Dr. J. L. Doob has been appointed research associate in theoretical statistics at Columbia University.

Mr. H. B. Huntley, of Rutgers University, has been appointed dean of the Union County Junior College, Roselle, N.J.

Dr. S. C. Kleene has been appointed research assistant at Princeton University.

Mr. T. L. Koehler has been promoted to an assistant professorship at Muhlenberg College.

Assistant Professor N. H. McCoy has been promoted to an associate professorship at Smith College.

Associate Professor H. F. MacNeish has been promoted to a professorship of mathematics at Brooklyn College.

Associate Professor V. S. Mallory has been promoted to a professorship of mathematics at the New Jersey State Teachers College, Montclair.

Mr. J. D. Mancill has been promoted to an assistant professorship at the University of Alabama.

Miss Thirza A. Mossman has been promoted to an assistant professorship at Kansas State College, Manhattan, Kansas.

Dr. A. F. Moursund has been promoted to an assistant professorship at the University of Oregon.

Associate Professor C. V. Newsom, of the University of New Mexico, has been promoted to a professorship of mathematics.

Dr. O. M. Norlie, formerly dean of the graduate school of Hartwick Seminary, Brooklyn, has been appointed professor of psychology at Luther College, Decorah, Iowa.

Dr. A. E. Staniland, of the University of Pittsburgh, has been promoted to to assistant professorship.

Dr. W. J. Trjitzinsky, of Northwestern University, has been appointed to an assistant professorship at the University of Illinois.

Assistant Professor Morgan Ward, of the California Institute of Technology, has been appointed to an assistant professorship at the Institute for Advanced Study.

The following appointments to instructorships in mathematics are announced:
University of Alabama: Mr. J. P. Gill;
Brooklyn College: Mr. Walter Prenowitz;
University of California (Berkeley): Dr. R. D. James;
Connecticut College for Women: Dr. Julia W. Bower;
Harvard University: Dr. M. R. Hestenes and Dr. Saunders MacLane (Peirce instructors);
Princeton University: Mr. A. H. Taub;
University of Rochester; Dr. G. B. Price;
University of South Dakota: Mr. W. E. Ekman;
Wellesley College: Dr. Ruth G. Mason;
University of Wisconsin: Mr. Henry Scheffé.

Dr. Pasquale Calapso, professor of calculus at the University of Messina, died May 4, 1934, at the age of sixty-two.

Mme Marie Curie, professor of general physics in the Faculty of Sciences of the University of Paris, and director of the Laboratoire Curie at the Institut du Radium, known for her work with her husband, Pierre Curie, leading to the discovery of radium, and for subsequent researches on radioactivity, died July 4, 1934.

Giulio Pittarelli, professor of descriptive geometry at the University of Rome, died March 3, 1934, at the age of eighty-two.

Dr. Heinz Prüfer, associate professor of mathematics at the University of Münster, is dead.

Associate Professor J. E. Stocker, of the department of mathematics at Lehigh University, died July 5, 1934, at the age of sixty.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

254. Professor A. A. Albert: *On cyclic fields.*

The author has recently given a construction of all cyclic fields of degree p^e over F of characteristic p. The theory of the structure of cyclic fields will now be made complete by his new results on the case where F has characteristic not p. The author proves that if F contains a primitive pth root of unity ζ, then a necessary and sufficient condition that Y of degree $p^{e-1}>1$ shall have cyclic over-fields Z of degree p^e is that $N_{Y|F}(\beta)=\zeta$ for some β in Y, and gives a construction of all such fields Z. The case where ζ is not F is then treated. (Received July 19, 1934.)

255. Professor A. A. Albert: *A greatest common divisor process for integers of a quadratic field.*

The author considers quadratic fields $F=R(m^{1/2})$ with m an integer having no square factors. If $m\equiv 2, 3 \pmod 4$, then any two integers of F have the form $A=\pi(a+bm^{1/2})$, $B=\sigma(c+dm^{1/2})$ where we may take $(\pi, \sigma)=(a, b)=(c, d)=1$ without loss of generality. Define μ to be the g.c.d. of $\pi(a^2-b^2m)$, $\sigma(c^2-d^2m)$, $\pi\sigma(ad-bc)$, and write $\mu=PQ$ where P is the product of all the prime factors of μ dividing π. Then there exists an integer ρ such that $b\rho\equiv a \pmod Q$, $d\rho\equiv c \pmod P$, and $\rho^2=\lambda\mu+m$. It is then proved that A and B have a g.c.d. in F if and only if the quadratic form $\lambda x^2+2\rho xy+\mu y^2$ represents 1 or -1. A similar result is obtained for the case where $m\equiv 1 \pmod 4$. (Received July 19, 1934.)

256. Professor A. A. Albert: *The principal matrices of a Riemann matrix.*

A Riemann matrix ω is a p by $2p$ complex matrix with an attached rational skew symmetric matrix C such that $\omega C\omega'=0$, $i\omega C\bar{\omega}'$ is positive definite. The matrix C is called a principal matrix of ω and it is important to know what are all principal matrices of ω in terms of a given C. In the present note I prove that the set of all principal matrices of ω is the set of all matrices AC where A ranges over all symmetric projectivities of ω with positive characteristic roots. More explicit formulas for A are given for the case where ω is a pure Riemann matrix of the first kind. (Received July 19, 1934.)

257. Dr. R. E. Basye: *Multiply connected sets.*

In a previous abstract (see this Bulletin, vol. 39, p. 862) the writer defined "simply connected set." In the present paper the natural extension of this definition to multiply connected sets of finite order is made. A necessary and sufficient condition that a connected subdomain of the plane be multiply connected of order n is that its complement have exactly $n-1$ bounded components. Among the applications is the following: In a plane S let N be a non-vacuous closed set, G any collection of closed sets, and H a countable subcollection (possibly vacuous) of G such that (1) the common part of each pair of elements of G is N, (2) every component of every element of $G-H$ intersects N, and (3) G^* is closed. If G^* separates a point A from a point B in S then G contains a countable subcollection K such that K^* intersects every compact continuum which contains $A+B$. (Received July 23, 1934.)

258. Dr. L. M. Blumenthal (National Research Fellow): *An application of a theorem of K. Gödel.*

Gödel has shown (Ergebnisse eines mathematischen Kolloquiums, Vienna, vol. 4 (1933), p. 16) that any quadruple of points in the euclidean space R_3 may be isometrically imbedded in the surface of a sphere. This theorem is applied to show that every quadruple of points of the three-dimensional spherical surface $S_{3,r}$ of radius r has this property also; that is, any four points of the $S_{3,r}$ may be imbedded isometrically in the surface $S_{2,r'}$ of some sphere of radius r'. (Received July 24, 1934.)

259. Dr. L. M. Blumenthal (National Research Fellow): *A decomposition of the spherical surface $S_{2,r}$ by means of tetrahedral quadruples.*

The points of the surface of a three-dimensional sphere of radius r, with the distance between two points defined as the geodesic (great circle) distance, form the spherical surface $S_{2,r}$. Four points of a semi-metric space form a tetrahedral quadruple provided the points are isometric with the vertices of a tetrahedron in the euclidean space R_3. The following theorem is proved: Corresponding to each triple of points p, q, r of $S_{2,r}$ there is a decomposition of $S_{2,r}$ into three sets of points: the set (x) for which p, q, r, x form a non-degenerate tetrahedral set; the set; (y) for which p, q, r, y form a degenerate tetrahedral quadruple; and the set (z) for which the points p, q, r, z are not congruent to four points of any euclidean space. If the points p, q, r lie on a great semi-circle, the set (y) is, of course, vacuous, while the set (x) consists merely of the points of the great circle containing p, q, r minus the arc $p'r'$, where p' and r' are the points diametral to p and r, respectively. (We suppose that q lies between p and r.) If p, q, r do not lie on a great semi-circle, the set (x) consists of a curve with two branches, bounding the sets (y) and (z). The nature of this curve is studied. (Received July 13, 1934.)

260. Professor H. R. Brahana: *Concerning the order of a linear transformation with coefficients in a $GF(p)$.*

Let the transformation be defined by the m-rowed square matrix X whose mth invariant factor is I_m. Then the period of the transformation is the same

as the exponent to which I_m belongs, mod p. There is also obtained an expression for the order in terms of the zeros of I_m. (Received July 30, 1934.)

261. Professor H. R. Brahana: *Metabelian groups and pencils of bilinear forms.*

We consider the metabelian groups $G=\{H, U\}$ of order p^{n+m} with centrals of order p^{n-k} in which H is a maximal invariant abelian subgroup of order p^n and type 1, 1, $\cdots$ and U is abelian and of type 1, 1, $\cdots$. If the order of the commutator subgroup of G is p^l the problem of classification of the groups G is equivalent to the problem of classification of k-parameter (homogeneous) families of bilinear forms in l variables y and m variables x under projective transformations on the x's, the y's, and the k parameters, the coefficients of the forms and of the transformations being residues mod p. The classification is carried out in detail for $k=2$, $m=4$, in which case $l=2, 3, \cdots, 8$. (Received July 12, 1934.)

262. Professor H. B. Curry: *Foundations of the theory of abstract sets from the standpoint of combinatory logic.*

This is a formulation of Zermelo's theory of abstract sets (Mathematische Annalen, 1908) based on the author's *Functionality in combinatory logic* (see abstract 36-9-324). The object is to remove the vagueness inherent in Zermelo's notion of "definiteness." Fraenkel has done this by introducing an ad hoc definition of function; the author's general notion of function will accomplish the same end. The more formal axioms—Paarung, Vereinigung, Potenzmenge, Aussonderung, and Ersetzung (as added by Fraenkel)—here take forms of which the following are typical: Axiom der Vereinigung: $\vdash FMMS$; Axiom der Aussonderung: $\vdash F_2(FM\overline{Pr})MMT$, where M stands for "Menge," $\overline{Pr}$ for "proposition," and S and T for operators to be defined. For primitive ideas we may take M and ϵ; but the development is simpler if we take M and an abstraction operator $A\phi$, such that $A\phi$ represents "the x's such that ϕx holds." Then ϵ, and the above S, T, etc., may be defined; the definition of T is $T\equiv[\phi, x]\cdot A([y](\phi y \,\&\, y\epsilon x))$ (that is, T is the operator such that, for each ϕ and x, $T\phi x$ is the set of y's for which $y\epsilon x$ holds and ϕy is true). Four additional axioms are necessary to express fundamentals. The Axiom der Auswahl and Axiom des Unendlichen are not affected. The resulting theory is more inclusive than Fraenkel's, less so than Zermelo's. (Received August 1, 1934.)

263. Mr. John Curtiss: *Interpolation in regularly distributed points.*

Walsh has proved the following theorem (this Bulletin, vol. 38 (1932), p. 290): Let $f(z)$ be an arbitrary function integrable (R) for $|z|=1$, and let polynomials $L_n(z)$ of respective degrees $n-1$, ($n=2, 3, \cdots$), be defined by the requirement of coinciding with $f(z)$ in the nth roots of unity; then the sequence $L_n(z)$ approaches the limit $[\int_{|z|=1}(f(t)/(t-z))dt]/(2\pi i)$ uniformly for $|z|\leqq r<1$. The present paper extends this theorem to the study of interpolation to an arbitrary function defined and integrable (R) on a Jordan curve which is subject to certain broad restrictions. The extension makes use of a particular

generalization of the nth roots of unity discovered by Fejér, which he termed a "regularly distributed" point set. The degree of convergence of the interpolating sequence is also investigated, both on and within the curve under consideration. The paper closes with certain extensions of the main theorems to the consideration of simultaneous interpolation to more than one function defined on one or more curves. (Received July 25, 1934.)

264. Mr. E. R. Elliot: *A mixed boundary-value problem for the heat equation.*

We consider a domain D bounded by two characteristics with ordinates h and e, and two arcs $x=X_i(y)$, X_i'' continuous. Segments of the curves $y=h+\delta$, $x=X_i(y)+\lambda_i$ are referred to as δ- and λ_i-displacements. We exhibit the unique function of the form $L(x, y)=\int U(x, y; \xi, h)d\beta(\xi)-2\sum\int U(x, y; X_i(\eta), \eta)dF_i(\eta)$, $U(x, y; \xi, h)=[4\pi(y-h)]\exp[-(x-\xi)^2/(4(y-h))]$, $y>h$, $=0$, $y\leqq h$, where the integration in the first integral is from $X_1(h)$ to $X_2(h)$ and in the second integral from h to y, which satisfies the equation $\partial^2u/\partial x^2-\partial u/\partial x=0$ within D, and satisfies the following boundary conditions: the line integral of $L(x, y)$ along a δ-displacement, as $\delta\to 0$, approaches $B(x_2)-B(x_1)$, and the line integral of $\partial L(x, y)/\partial x$ along a λ_i-displacement, as $\lambda_i\to 0$, approaches $G_i(y_2)-G_i(y_1)$, where $B(x)$, $G_i(y)$ are preassigned functions of limited variation, $B(x)$ being continuous at the end points with regular discontinuities elsewhere, $G_i(y)$ continuous at h and continuous from the left elsewhere. The methods employed are also shown to apply to a problem previously discussed by F. A. Dressel (American Journal of Mathematics, vol. 55 (1933), p. 649) and a simpler solution is thereby obtained. (Received July 21, 1934.)

265. Professor O. J. Farrell: *On approximation by polynomials to a function analytic in a simply connected region.*

In a previous paper (this Bulletin, abstract 40-3-152) the writer failed to mention a certain hypothesis to be put upon the region R laid down in the results (1) and (2) of that paper. The description of R should be amended to read as follows: "a finite simply connected region R whose boundary B is such that all the points of the extended plane not on $R+B$ form a single region whose boundary is precisely B." The present paper extends both results (1) and (2) of the earlier paper to an arbitrary limited simply connected region G whose boundary is also the boundary of an infinite region. Neither result can be extended to a region whose boundary is wholly arbitrary. In fact the region G laid down in the present paper is the most general region such that (1) holds for an arbitrary function $f(z)$ analytic and bounded therein. Examples of regions and functions for which (2) does not hold are also given. (Received July 16, 1934.)

266. Professor O. J. Farrell: *On the expansion of harmonic functions in series of harmonic polynomials belonging to a simply connected region.*

Earlier results (this Bulletin, abstract 36-3-71) on the expansion in harmonic polynomials of a function harmonic in an arbitrary finite Jordan region

are extended to an arbitrary limited simply connected region whose boundary consists wholly of simple boundary points and is also the boundary of an infinite region. This extension is accomplished through application of certain results by the writer on conformal mapping (American Journal of Mathematics vol. 54 (1932), pp. 571–578). (Received July 16, 1934.)

267. Professor L. R. Ford: *On properties of regions which persist in the subregions bounded by level curves of the Green's function.*

Let the circle $|z|<1$ be mapped conformally on a region S in the w-plane, the origin going into the origin; and let S_r be the map of $|z|<r<1$. Let $\theta(w_1, \cdots, w_n)$ be analytic when $w_1, \cdots, w_n$ range over S and $\theta(0, \cdots, 0)=0$. The following theorem is proved by an application of Schwarz's lemma: *If S has the property that when $w_1, \cdots, w_n$ are in S so also is $w_0=\theta(w_1, \cdots, w_n)$, then S_r likewise has this property.* Special cases are the known theorems that if S is convex so also is S_r [here $\theta=tw_1+(1-t)w_2$]; also if S is star-shaped from the origin so also is S_r. (Received July 30, 1934.)

268. Professor Philip Franklin: *On Minkowski's definition of length and area.*

For convex surfaces in three-space, Minkowski gave a definition of area based on volume. We here show that his method applies to any sufficiently regular surface. It thus gives a simple definition of surface area. (Received July 25, 1934.)

269. Professor Philip Franklin: *A six-color problem.*

For surfaces topologically distinct from the sphere, an argument of Heawood gives a number of colors sufficient for any map. This number is also an upper limit to the number of regions mutually in contact, so that whenever an example of this is at hand, the map coloring problem is solved. Such examples are known for several two-sided surfaces. In this note we show that for the one-sided surface of characteristic zero, while the Heawood number is seven, the maximum number of neighboring regions is in fact six, so that the map coloring problem for this surface is unsolved. (Received July 25, 1934.)

270. Mr. Bernard Friedman: *Sufficient conditions for the solvability of some Pellian equations.*

In the present paper there is discussed the equation $x^2-py^2=B$, where p is a prime. Lagrange's method for solving the equation is supplemented by introducing a further condition on p. This condition on p when combined with the necessary conditions on B and p will be sufficient for the solvability of the above equation. The condition which p will be assumed to satisfy is as follows. For every odd prime $q<p^{1/2}$ of which p is a quadratic residue, integers (x, y) exist satisfying $x^2-py^2=(-1)^{(q-1)/2}q$. Then it is proved that if $B=2^eN$ where N is an odd positive or negative integer and $e=0$ or $\geqq 3$ if $p=8n+1$, $e=0$ or 1 if $p=8n+3$, $8n+7$, $e=0$ or 2 if $p=8n+5$, and if B and p are quadratic residues of each other, there exists a solution in integers of $x^2-py^2=B$. For example, the equation $x^2-67y^2=B$ has integral solutions if and only if B and 67 are quad-

ratic residues of each other and if B does not contain the factor four. (Received July 25, 1934.)

271. Professor J. J. Gergen: *Note on linear operations in functions of bounded variation.*

The object here is to obtain a characterization of linear operators on functions of bounded variation. Let V be the set of functions $x(t)$ which vanish for $t=0$ and are of bounded variation for $0 \leqq t \leqq 1$. Let the operator $L(x)$ be defined over the space V and be linear there, where the norm $\|x\|$ is defined as $\int_0^1 |dx|$. We prove first that there corresponds to each x a function $K_x(t)$ such that (a) $K_x(t)$ is x-measurable and satisfies $|K_x(t)| \leqq$ norm of L for $0 \leqq t \leqq 1$, (b) $L(x) = \int_0^1 K_x(t)dx$, (c) for any two functions x, y, of V, $K_x(t) = K_y(t)$ except possibly on a set $E_{x,y} + E_{y,x}$ where $E_{x,y}$ is of x-measure 0 and $E_{y,x}$ of y-measure 0. As a corollary there follows the result somewhat complementary to F. Riesz's fundamental theorem, that, if V_1 is any separable subspace of V, there exists a $K(t)$ such that $K(t)$ satisfies (a) and $L(x) = \int_0^1 K(t)dx$ for all x in V_1. (Received August 1, 1934.)

272. Professor B. P. Gill: *Zeroless solutions of systems of linear equations in a finite field.*

Let f_i, $(i=1, 2, \cdots, m)$, be homogeneous linear functions of $x_1, x_2, \cdots, x_k$ in the $GF[p^n]$; there is studied in this paper the number σ of solutions in the field of the system of simultaneous equations $f_i=0$ such that no x_j has the value zero. Of the p^{mn} possible linear combinations of the f's with coefficients in the field, let ν_h contain precisely h x's with coefficients not zero. Then $\sigma = p^{-mn} \cdot (p^n-1)^k \sum_{h=0}^{k} \nu_h (1-p^n)^{-h}$. (Received August 1, 1934.)

273. Dr. M. R. Hestenes (National Research Fellow): *The problem of Bolza in the calculus of variations in parametric form.*

In the present paper we give a set of sufficient conditions for a minimum for the problem of Bolza in parametric form. These sufficient conditions have the same generality as those given by the author for the non-parametric case (abstract 40-3-141) in that no normality assumptions are needed. Of particular interest is the treatment of the second variation. We give two methods of studying the second variation. The first is a generalization of a method due to Weierstrass and is particularly adaptable to the present paper. The second method is essentially that of Bliss, and has been used with some modification by Morse, Graves, and others. It consists of introducing a new side condition to the accessory minimum problem. The side condition here used changes the order of anormality of the accessory minimum problem and thus avoids many difficulties which otherwise seem to arise. (Received July 28, 1934.)

274. Professor T. R. Hollcroft: *On nets of algebraic surfaces.*

The jacobian curve J of a net of algebraic surfaces contains all the singularities and contacts of surfaces of the net. The binodes and the pairs of conic nodes of surfaces, and the points of stationary contact and the pairs of points of simple contact of pencils of surfaces of the net lie at definite, but ordinary,

points of J. These points of J are located by immersing the net in a web of surfaces. The characteristics of a net of surfaces with basis curves are obtained. The Steinerian curve of a net of surfaces is defined and its projection on a plane is shown to be the reciprocal of the branch-point curve of the transformation established by the (1, 1) correspondence between the surfaces of the net and the lines of a plane. (Received August 1, 1934.)

275. Professor R. E. Langer: *On the asymptotic solutions of ordinary differential equations, with reference to the Stokes phenomenon about a singular point.*

The asymptotic representations of the solutions of a differential equation $u''+\{\lambda^2\phi(z)+\psi(z)\}u=0$, for large values of the parameter λ, are known to be subject to the Stokes phenomenon in any domain of the variable which includes a point at which the coefficient $\phi(z)$ becomes zero. The theory and formulas associated with this phenomenon have previously been given for the case in which the coefficient $\psi(z)$ remains bounded. The present paper extends this theory to admit in $\psi(z)$ a pole of the first or second order. It is shown that the presence of such a pole alone engenders the Stokes phenomenon, the latter being then quantitatively determined by the value $(z-z_0)^2\psi(z)$. If $\phi(z)$ and $1/\psi(z)$ vanish together, the phenomenon is determined by both coefficients jointly. The theory here given is applicable to the asymptotic representations of a number of classically important functions, including the ordinary and the associated Legendre functions, the Laguerre polynomials, and the Mathieu functions of higher order. (Received July 26, 1934.)

276. Mr. Norman Levinson: *On the vanishing of a function over an interval.*

Using the Wiener-Paley theorem on the vanishing of a function on the half-line, a condition depending on the behavior of the absolute value of the Fourier transform at infinity is given which is sufficient for the non-vanishing of a function in an interval, no matter how small. This condition is independent of the existence of derivatives, and is hence somewhat different in type from conditions of quasi-analyticity. (Received August 1, 1934.)

277. Mr. Norman Levinson: *A theorem on the magnitude of Fourier transforms.*

In connection with the statement of Wiener that a function and its Fourier transform cannot both be very small at infinity, the theorem will be proved that if $g(u)$ is the Fourier transform of $f(x)$, if $f(x)=O(\exp(-|x|^p(\log|x|)^a))$ and if $g(u)=O(\exp(-|u|^q(\log|u|)^b))$, then if $1/p+1/q=1$ and if a and b exceed certain quantitites depending on p and q, $f(x)\equiv g(u)\equiv 0$. In the case $p=\infty$, not included in the above, and already treated by Ingham, a very simple proof of the corresponding theorem may be given, using Wiener's and Paley's theorem on functions vanishing on a half-line. (Received August 1, 1934.)

278. Dr. Hans Lewy: *A priori limitations for solutions of elliptic Monge-Ampère equations.*

Let $x(u, v)$ be an analytic function in $u^2+v^2 \leqq 1$, and a solution of a Monge-Ampère equation with analytic coefficients, which is elliptic for the given x. Let M_1, M_2, M_3 be bounds for the absolute values of x, its first, and its second derivatives, respectively. Then there exist bounds for partial derivatives of x of every order, which do not depend on the particular solution x, but merely on the equation and the constants M_1, M_2, M_3. This theorem has important applications in differential geometry in the large. (Received July 31, 1934.)

279. Professor N. H. McCoy: *On certain rings and differential ideals.*

Let K be a commutative ring with unit element in which each ideal has a finite ideal basis. The primary purpose of this paper is to study rings of the form $K[\alpha, \beta]$ where α and β are commutative with elements of K and $\alpha\beta-\beta\alpha$ is commutative with both α and β. A special case of interest is the algebra of quantum mechanics, in which there exist elements p, q such that $pq-qp=1$. A characterization of rings $K[\alpha, \beta]$ is given in terms of the notion of differential ideal in a certain commutative ring. Special attention is given to the case in which K is a field and $K[\alpha, \beta]$ is a finite algebra over K. (Received July 21, 1934.)

280. Dr. Saunders MacLane: *Some unique separation theorems for graphs.*

Since a graph is formed by any number of vertices arbitrarily joined in pairs by edges, it is desirable to have some method of constructing complex graphs from simpler ones. The separation of graphs by subgraphs is one such procedure. Whitney, in the Transactions of this Society, vol. 34 (1932), p. 339, has considered the separation of graphs by single vertices. More generally, any connected graph G is *separated* by a subgraph H if the removal of all the vertices and edges of H disconnects G into two or more pieces. The question of uniqueness at once arises: if a graph is separated as far as possible, is the result independent of the order of the separations? This paper considers this question for separations by chains and by cycles. The successive separation by chains is unique if each chain used is shorter than any other chain joining the same extremities. A similar uniqueness theorem for "shortest" cycles is established. The proofs require investigation of the various symmetrical figures in which two short subgraphs can intersect. Several specimen graphs show that the requirement of "shortness" for unique separation is a natural one. (Received July 30, 1934.)

281. Dr. Saunders MacLane: *A classification of finite distributive lattices.*

The presence of common formal properties for the two dual operations "least common multiple" and "greatest common divisor" suggests the abstract study of all systems or "lattices" having these properties. One natural problem is the investigation of the possible sublattices in a given lattice. For finite distributive lattices it is sufficient to consider two special types of sublattices: the "normal" sublattices and the sublattices which contain the unit element of the original lattice. The normal sublattices are somewhat analogous

to normal subgroups; their structure can be easily exhibited in terms of their irreducible elements. Furthermore, these normal sublattices show that any lattice can be split up into two exhaustive and mutually exclusive sublattices. The second type of sublattices, those containing the original unit element, have a more complex structure, but their irreducible elements can be related systematically to the irreducible elements of the original lattice. (Received July 30, 1934.)

282. Professor M. L. MacQueen: *A projective generalization of metrically defined associate surfaces.*

In the metric differential geometry of surfaces in ordinary space, two surfaces are said by Bianchi to be associate if the tangent planes at corresponding points are parallel and if the asymptotic curves on either surface correspond to a conjugate net on the other. The purpose of this paper is to develop a projective generalization of the relation of associateness of surfaces. The first problem is to provide a projectively defined substitute for the metric property of parallelism of surfaces. This provision is made by employing a projective generalization of euclidean parallelism of surfaces developed in the author's thesis and summarized briefly in this paper. One of the well known transformations of surfaces, namely, the fundamental transformation, is employed in constructing a projective analogue of the property of metric parallelism of surfaces. After formulating a definition of projectively associate surfaces, their properties and relations are studied in some detail. A more general type of associateness called modified projective associateness is introduced and is briefly studied. (Received July 15, 1934.)

283. Professor Morris Marden: *On the zeros of the derivative of a rational function.*

In this paper the author redetermines by a new method the locus of the zeros of the derivative of the rational function $f(z)=(z-z_0)^{m_0}(z-z_1)^{m_1}\cdots\cdot(z-z_p)^{m_p}$, the m_j being real or complex, when each point z_j varies independently over a given circular region Z_j. He finds again the locus to be bounded by a certain p-circular $2p$-ic curve, which reduces if $\sum m_j=0$ to a $(p-1)$-circular $2(p-1)$-ic curve. The new treatment, which is much shorter and simpler than the one given in the author's doctoral thesis (Transactions of this Society, vol. 32 (1930), pp. 81–109), is based upon the lemma that, if the points u and v independently describe given circular regions, the point $w=u+v$ also describes a circular region. The results also include a determination of the locus of the zeros of any linear combination of $f(z)$ and its derivative. (Received July 31, 1934.)

284. Professor G. A. Miller: *Groups in which the squares of the elements are dihedral groups.*

Various categories of groups can be conveniently studied by means of their subgroups composed of the elements which are squares of other elements of the group. In the present article the author considers the category in which these subgroups are dihedral. There is only one abelian dihedral group and this is the

case which presents the greatest difficulties as regards the category of groups under consideration. The paper has been accepted for publication in the Transactions of the Society. (Received July 23, 1934.)

285. Dr. E. P. Northrop: *An operational solution of the Maxwell field equations.*

This paper is concerned with the solution of the Maxwell electrodynamic field equations in the case of the simplest type of field, that is, free space, containing no charges or currents. By a proper choice of units, these equations can be expressed in the form $\nabla \cdot \boldsymbol{E}=0$, $\nabla \cdot \boldsymbol{H}=0$, $\nabla \times \boldsymbol{E}=-\partial \boldsymbol{H}/\partial t$, $\nabla \times \boldsymbol{H}=\partial \boldsymbol{E}/\partial t$, where $\boldsymbol{E}$ and $\boldsymbol{H}$ are respectively the electric and magnetic intensities. If we denote by $\boldsymbol{V}$ the six-component vector $(E_x, E_y, E_z, H_x, H_y, H_z)$, the two curl equations can be written as the single equation $\partial \boldsymbol{V}/\partial t=i\boldsymbol{H}\boldsymbol{V}$, where H is a certain matrix differential operator. The solution of this equation is formally $\boldsymbol{V}=e^{i\boldsymbol{H}t}\boldsymbol{V}_0$ where $\boldsymbol{V}_0$ characterizes the initial state of the field. The operator $e^{i\boldsymbol{H}t}$ is determined in a rigorous manner by means of the operational calculus, as developed in M. H. Stone's treatise *Linear Transformations in Hilbert Space*. The significance of the divergence conditions is briefly discussed. The work results in a general integral representation for the vector $\boldsymbol{V}$ characterizing the electromagnetic field at any point and at any time. (Received July 19, 1934.)

286. Mr. L. B. Robinson: *A singular solution of a functional equation.*

Consider a special Izumi equation (1) $u'(x)=P_n(x)u(x/2)+Q_n(x)$. Where P_n and Q_n are polynomials of degree n. Write $u(x)\equiv v(1/x)\equiv v(\xi/2)$. The above equation becomes (2) $v'(\xi/2)\xi^{n+2}=R_n(\xi)v(\xi)+S_n(\xi)$, $R_n(0)\neq 0$. Equation (1) has a solution everywhere holomorphic. Equation (2) has a solution holomorphic so long as $R_n\neq 0$. In the neighborhood of $x=0$, $\xi=\infty$, v is uniform or non-uniform. If uniform, anessential singularity exists. Then by the Picard-Weierstrass theorem on essential singularities, it cannot be identified with u. If v is non-uniform, it cannot, ipso facto, be identified with u. So at last an example of a singular solution has been found. To avoid using the law of the excluded middle, the author by resting on Liouville's theorem has proved the same result. (Received July 14, 1934.)

287. Mr. R. Q. Seale: *A simple proof of Minkowski's theorem concerning non-homogeneous linear forms.*

In this paper a new proof is given of Minkowski's theorem that if $\Delta=A_1B_2-A_2B_1=1$, integers x, y always exist such that $|(A_1x+B_1y+C_1)(A_2x+B_2y+C_2)|\leq\frac{1}{4}$; it is further proved that if A_1/B_1 is irrational, $|A_1x+B_1y+C_1|$ can, at the same time, be made arbitrarily small. The proof is simple, direct, and is based on no ideas more advanced than the elementary properties of convergents. (Received July 30, 1934.)

288. Professor I. M. Sheffer: *A local solution of the difference equation $\Delta y=f$ and of related equations.*

The equation $\Delta y=f$ has been studied from a number of viewpoints. The

more important are (1) with respect to the asymptotic character of $f(x)$ at infinity; (2) concerning proofs that if $f(x)$ is an entire function (meromorphic function) then an entire (meromorphic) solution exists. So far as we are aware, however, there has been no treatment for an arbitrary analytic function. The present paper aims to fill this gap. We obtain a local existence theorem. The method consists in the application of the Borel integral and its inverse. The same method, leading to the same conclusions, applies to related equations such as $\sum_1^n \alpha_i y(x+c_i)=f$, $\sum_1^n \alpha_i y^{(i)}(x)=f$, and also to equations involving certain more general differential operators with constant coefficients. (Received July 28, 1934.)

289. Dr. Mildred M. Sullivan (National Research Fellow): *On the partial derivatives of harmonic functions as functionals of the determining boundaries.*

In this paper, the partial derivatives of harmonic functions determined in regions of space by their values on the boundaries of the regions are considered as functionals of the boundaries, and conditions for the continuity of these functionals with respect to the boundaries are obtained. (Received July 30, 1934.)

290. Dr. Henrietta Terry: *Abelian subgroups of the I-group of the abelian group of order p^n and type* 1, 1, · · · .

This paper determines a set of maximal abelian subgroups of a Sylow subgroup I_p of I in which every abelian subgroup of I_p is contained. Methods are developed for the classification of subgroups of order p^2 of I_p, and the classification is carried out for some of the earlier cases. (Received July 13, 1934.)

291. Professor J. V. Uspensky: *On the expansion of the remainder in the Newton-Cotes formula.*

The author shows that the remainder of the Newton-Cotes quadrature formula can be expanded into series quite similar to the classical Euler-Maclaurin series. The general proof in this case is more difficult than in the case of the Gaussian formula and requires an elaborate study of some properties of Cotes coefficients. (Received July 30, 1934.)

292. Professor Louis Weisner: *Irreducibility of polynomials of degree n which assume the same value n times.*

The polynomial $f(x)=ax(x-t_1)\cdots(x-t_{n-1})\pm k$, in which a, k, $t_1, \cdots, t_{n-1}$ are positive integers and the t's distinct, is proved to be irreducible in the field of rational numbers if at least one of the n inequalities $a>2^n k^2$, $t_i>4nk$, $(i=1, \cdots, n-1)$, is satisfied. Every polynomial of degree n which assumes the same value for n distinct integral values of the variable is equivalent, under simple transformations which do not affect the reducibility of the polynomial, to one of the form of $f(x)$. Hence only a finite number of non-equivalent reducible polynomials of degree n exist which assume a given integral value for n different integral values of the variable. (Received July 27, 1934.)

293. Dr. Hassler Whitney: *Analytic approximations to manifolds.*

Let M be a point set in n-space E_n. We shall say M is an *m-manifold in regular position* in E_n if the following is true: (1) There are $n-m$ continuous vector functions $f_i(p)$ defined over M, such that $f_1(p), \cdots, f_{n-m}(p)$ are linearly independent for each p. (2) For each p in M there is a neighborhood U of p and an $\epsilon>0$ such that (a) U is an m-cell, (b) if p_1 and p_2 are in U, then the line through p_1 and p_2 makes an angle $\geqq\epsilon$ with the $(n-m)$-plane through p_1 determined by $f_1(p_1), \cdots, f_{n-m}(p_1)$. Certain classes of manifolds are shown to have this property. Theorem: *Given M as above and a continuous positive function $\phi(p)$ defined over M, there is an analytic manifold M' homeomorphic with M, such that corresponding points p and p' of M and M' are within a distance $\phi(p)$ of each other.* (Received July 23, 1934.)

294. Dr. Hassler Whitney: *On the abstract properties of linear dependence.*

Let M be a set of elements $C_1, \cdots, C_n$, and let any subset either be or be not "(linearly) independent." Suppose (1) any subset of an independent set is independent, and (2) if $C_1, \cdots, C_m$ and $C_1', \cdots, C_{m+1}'$ are independent, then for some i, $C_1, \cdots, C_m, C_i'$ are independent. Then we shall call M a *matroid*. Examples of sets of elements forming a matroid are the columns of a matrix, and the arcs of a linear graph (a set of arcs being independent if it contains no circuit). Other sets of axioms for matroids are given, in terms of bases, circuits, rank. Properties of matroids (especially of non-separable and dual matroids) are found corresponding to theorems on graphs in a paper by the author (Transactions of this Society, 1932). Dual matrices have a simple geometrical interpretation. Matroids form a larger class than matrices, as an example of a matroid with nine elements shows. (Received July 23, 1934.)

295. Mr. L. B. Robinson: *The equation of Izumi and a contour problem.*

Izumi (Tôhoku Mathematical Journal, 1929) has considered the equation (1) $u'(x)-\phi\equiv u'(x)-a(x)u(\omega(x))-c(x)=0$. He demonstrates the existence of a solution in the region $|x|<\rho$, $|\omega(x)|<\rho$. His method seems to fail when we consider the case (2) $u'(x)-f=u'(x)-a(x)u(\omega(x))-b(x)u(x)-c(x)=0$ under the hypothesis that when $|x|\leqq 1$, then $a(x)$, $b(x)$, $c(x)$, and $\omega(x)$ are regular and also $|\omega(x)|\leqq 1$. But the author replaces (2) by its equivalent $u'(x)-f=0, \cdots, u^n(x)-[f^{(n-1)}]=0$. The brackets signify that all derivatives which appear in the f have been eliminated. The author obtains a solution which is valid in a domain D and on its *contour*. He can extend his results to the case $(a\partial/\partial x)^n u(x)-f=0$. (Received August 2, 1934.)

296. Professor P. A. Smith: *The fundamental groups of group manifolds.*

The principal result of this paper is that the maximum number of linearly independent elements in the (abelian) fundamental group of an r-parameter group manifold is at most r. (Received August 4, 1934.)

297. Professor Otto Szász: *Convergence properties of Fourier series.*

Let (B) be the class of series $\sum c_n$ such that the series $\sum (c_n - |c_n|)$ is "slowly oscillating below." The present paper deals with various convergence and boundedness properties (uniform convergence and uniform boundedness) of Fourier cosine and sine series whose coefficients belong to the class (B). The results of the paper can be extended to almost periodic functions and to Fourier integrals. (Received August 6, 1934.)

298. Professor W. D. Baten: *A formula for finding the skewness of the distribution made up of a finite number of samples.*

This article develops a formula for the skewness of a frequency distribution made up of the variates in a finite number of samples in terms of the numbers of variates in each sample, the means, standard deviations, and skewness of the various samples. This sum formula is obtained by expressing the characteristics of each sample in terms of the fundamental summations of the variates. At the close of the article there is an illustration indicating the use of the formula and an effective way of carrying out the computations. (Received August 10, 1934.)

299. Mr. Garrett Birkhoff: *Integration of functions with values in a Banach space.*

The point sets of any vector space constitute a "vectoroid space" in which only the postulate $x+(-x)=0$ fails. Relative to this space, convex hulls define a homeomorphism. Based on these notions, and the "unconditional" or commutative convergence of Orlicz, a process of integration for functions with values in a Banach space is created which is effectively more powerful than the method of Bochner. As usual, definite integrals are additive; but the space of integrable functions is not in general complete. If the Banach space is of finite dimensions, our method is in effect Lebesgue's. (Received August 4, 1934.)

300. Dr. L. A. Dye: *The number of trisecants of a space curve of order m which meet an i-fold secant.*

By means of a correspondence, the number of trisecants of a space curve of order m which meet an i-fold secant is determined to be $(m-2)[h-m(m-1)/6] - i(h-m+2)+i(i-1)\ (i-2)/6$. (Received August 15, 1934.)

301. Professor Marston Morse and Mr. G. B. Van Schaack: *The boundary conditions in the critical point theory.*

Morse has previously treated the problem of the critical points of a real analytic function f of n variables with no restrictions on the nature of the critical points or the function on the boundary. However, the function f was required to have a positive normal derivative on the boundary. In the present paper this requirement is replaced by the condition that f have no critical points on the boundary. The results are obtained under hypotheses which include cases broader than the analytic case. (Received August 7, 1934.)

302. Professor Marston Morse and Mr. G. B. Van Schaack: *The critical point theory under general boundary conditions.*

The present paper has two purposes. The first is to simplify as far as possible the treatment of critical points of non-degenerate functions of n variables. The second is to extend the earlier treatment of Morse to the case of general boundary conditions. The boundary condition that the normal derivative f_N of f be positive on the boundary is replaced in the general boundary conditions by the requirement that f have no critical points on the boundary, the boundary function ψ being non-degenerate. The function f is replaced by a function F which is identical with f except neighboring the boundary and which has a normal derivative F_N positive on the boundary. The new function F has as additional critical points a critical point of index k neighboring each critical point of index k of ψ at which $f_N < 0$. (Received August 7, 1934.)

303. Professor Marston Morse: *Sufficient conditions in the problem of Lagrange without assumptions of normalcy.*

Sufficient conditions in the ordinary Lagrange problem in a form which involves the Jacobi or conjugate point condition have not previously been established without assumptions of normalcy, although considerable effort has been made to reduce such assumptions. The present paper establishes such conditions for the first time. (Received August 7, 1934.)

304. Professor G. C. Evans: *Short proof of Kellogg's lemma.*

Let T be an unbounded domain whose frontier t is a closed bounded, reduced set; let the complement of T be CT; let a conductor potential of t be $V(M)$. A short proof of Kellogg's lemma, that t contains at least one point regular with respect to T, is contained in the following statements: (a) A sufficient condition that Q be regular is that $V(Q)$ have the value 1 (Vasilesco). (b) The upper bound of $V(M)$ is 1 in any neighborhood of any point of t (Kellogg, Vasilesco, Bouligand). (c) If $V(M)$ is continuous at Q of t, for M on t, it is then continuous at Q for arbitrary approach of M (Evans). (d) $V(M)$ is of Baire class 1 in CT, and is therefore punctually discontinuous (Baire), so that there are points Q of t where $V(M)$ is continuous for M on t. (Received August 17, 1934.)

305. Dr. Hillel Poritsky: *On the nature of the "reflection" in a plane boundary corresponding to the boundary condition* $\partial u/\partial n + \text{const.}\ u = 0$.

The "reflection" or analytic continuation of a harmonic function u across a plane boundary along which either of the conditions $u=0$, $\partial u/\partial n = 0$ holds, is well known. In this note the reflection across a plane boundary $x=0$ along which the boundary condition $\partial u/\partial x + au = 0$, $a = \text{const.} \neq 0$, holds, is investigated. It is shown that the Green's function for this condition, with a pole at P in $x>0$, when continued analytically to $x<0$ has the following singularities: (a) a positive image at P', the reflection of P in $x=0$; (b) an exponential trail of negative images along the line through P_1P', from P' to $x=-\infty$. Similar

"reflections" are established for the heat conduction and certain other partial differential equations. (Received September 5, 1934.)

306. Dr. W. T. Reid: *The theory of the second variation for the non-parametric problem of Bolza.*

The principal result obtained in this paper is as follows: If E_{12} is an extremal arc for a non-parametric problem of Bolza having multipliers $\lambda_0=\text{const.}$, $\lambda_\alpha(x)$ with which it satisfies the strengthened Clebsch condition, and there is no point on E_{12} conjugate to the point 1, then there exists a family of mutually conjugate solutions of the accessory equations whose determinant is non-zero along E_{12}. In view of this result, the sufficiency theorems of Morse (Annals of Mathematics, vol. 33 (1932), pp. 261–274) for an extremal E_{12} which is normal on x_1x_2 are valid without the assumption of normality on sub-intervals x_1x_3 $(x_1<x_3<x_2)$ that has been used by each of these authors. Similarly, the sufficiency theorems of Bliss and Hestenes (Transactions of this Society, vol. 35 (1933), pp. 305–326 and pp. 479–490) for the problem of Mayer are valid with correspondingly weakened hypotheses. Hestenes (abstracts 39–5–124, 39–7–213, and 40–3–141) has obtained sufficiency theorems in the problem of Bolza assuming no normality conditionsa side from the existence of multipliers of the form $\lambda_0=1$, $\lambda_\alpha(x)$, by modifying the usual form of the Mayer condition. The results of this paper enable one to state these theorems with the Mayer condition phrased in either the form used by Morse or that used by Bliss. (Received September 5, 1934).

307. Mr. W. H. Ingram: *Dynamics of the homopolar generator.*

The slip-rings are more than a means to avoid twisting conductors in a Maxwell net, and constitute dynamically essential elements in the machine. The voltage at the brushes is given by Lagrangean multipliers, and in steady running with constant field current I the internal e.m.f. for the axial type is given by $(4/\pi)Im(d\theta/dt)$ where m is the inductance mutual to field circuit and slip-ring. The physical coordinates are true, as in all slip-ring machines, and the space of the trajectories, as for all machines of any kind, is Riemannian. Null-functions of the form $\lambda_i(\sum A_{ij}(dq_j/dt))$ are added to the activity function in a transformation of variable to yield the transformed Lagrangean multipliers. (Received September 7, 1934).

308. Professor E. T. Bell: *Ternary arithmetical identities.*

The addition theorems for the elliptic theta functions imply a set of precisely four identities, none of which can be derived from others by transformations of the first and second orders, which are bilinear in theta functions and doubly periodic functions of the second kind. The arithmetical equivalents of this set refer to all representations of positive integers in the form x^2+yz, with y, z non-negative; the arithmetical function involved is $L(u, v, w)$, which is finite and single-valued for integer values of u, v, w, and is an alternating function of u, v, w. One of the four is equivalent to the fundamental identity of Uspensky; the other three are new. The set exhausts the identities of this type implied by the addition theorems for the thetas. (Received August 31, 1934.)

309. Professor A. F. Moursund: *On summation of derived series of the conjugate Fourier series.* II.

This paper is an extension of the author's paper *On summation of derived series of the conjugate Fourier series* (abstract 39–9–256, this Bulletin). Three theorems concerning the $N_{z,p}$ summability of the rth derived series of the conjugate Fourier series are given. These theorems and a theorem of the earlier paper yield, by specialization of the $N_{z,p}$ method, theorems for the Bosanquet-Linfoot and Cesàro methods. The case $r=0$ gives four theorems, three well known and one new, for the Cesàro summability of the conjugate Fourier series. (Received August 7, 1934.)

ERRATUM

Volume 40, page 388, abstract No. 204 (by Sister Mary Cleophas Garvin, S.N.D.): the formula for the series in the first line, which was printed as $\sum_{n=1}^{\infty} a_n z^{\lambda_n}/(1-z^{\mu_n})$, should read $\sum_{n=1}^{\infty} a_n z^{\lambda_n}/(1-z^{\mu_n})$.

NEW PUBLICATIONS

PART I. PURE MATHEMATICS

BACHELARD (G.). Le nouvelle esprit scientifique. (Nouvelle Encyclopédie Philosophique.) Paris, Alcan, 1934.

CARNAP (R.). Logische Syntax der Sprache. (Schriften zur Wissenschaftliche Weltauffassung, Band 8.) Wien, Springer, 1934. 12+274 pp.

CAZALAS (E.). Carrés magiques au degré *n*. Séries numérales de G. Tarry avec un aperçu historique et une bibliographie des figures magiques. Paris, Hermann, 1934.

CERCONE DE LUCIA (C.). See MIRONE (T.).

DUBISLAV (W.). Naturphilosophie. Berlin, Junker und Dünnhaupt, 1933. 172 pp.

DURELL (C. V.) and ROBSON (A.). Higher certificate calculus. London, Bell, 1934. Pp. 11+241–368.

EINSTEIN (A.). Comment je vois le monde. Paris, Flammarion, 1934.

DE FERMAT (P.). Abhandlungen über Maxima und Minima (1629). Aus dem Lateinischen übersetzt und mit Anmerkungen versehen von M. Miller. (Ostwalds Klassiker der Exakten Wissenschaften.) Leipzig, Akademische Verlagsgesellschaft, 1934. 50 pp.

GODEAUX (L.). Leçons de géométrie infinitésimale. Liége, Imprimerie-Lithographie Pholien, 1933. 158 pp.

——— Les surfaces algébriques non rationnelles de genres arithmétique et géométrique nuls. Paris, Hermann, 1934.

——— La théorie des surfaces et l'espace réglé. Paris, Hermann, 1934. 36 pp.

HASSE (H.). Aufgabensammlung zur höheren Algebra. (Sammlung Göschen.) Berlin, de Gruyter, 1934. 175 pp.

JANET (P.). Paul Painlevé. Notice lue à l'Assemblée générale de l'Association de secours des anciens élèves de l'Ecole Normale Supérieure, le 14 janvier 1934. Paris, 1934.

JESSOP (T. E.). A bibliography of George Berkeley. With an inventory of Berkeley's manuscript remains by A. A. Luce. Oxford, 1934. 16+99 pp.

LINDELÖF (E.). Einführung in die höhere Analysis. Nach der ersten schwedischen und zweiten finnischen Auflage Deutsch herausgegeben von E. Ullrich. Leipzig, Teubner, 1934. 10+526 pp.

LUCE (A. A.). See JESSOP (T. E.).

MENGER (K.). Moral, Wille und Weltgestaltung. Grundlegung zur Logik der Sitten. Wien, Springer, 1934. 4+143 pp.

MILLER (M.). See DE FERMAT (P.).

MIRONE (T.) e CERCONE DE LUCIA (C.). Nuove tavole numeriche e norme per l'uso dei logaritmi e del regolo calcolatore. Genova, A. Colombo, 1933. 118 pp.

NOVIKOV (P.). See ZEUTHEN (H. G.).

POOR (V. C.). Analytical geometry. New York, Wiley, and London, Chapman and Hall, 1934. 5+244 pp.

PRASAD (G.). Some great mathematicians of the nineteenth century. II. Benares, Benares Mathematical Society, and Leipzig, Koehler's Antiquarium, 1934. 18+324 pp.

PUIG ADAM (P.). See REY PASTOR (J.).

REY PASTOR (J.) y PUIG ADAM (P.). Elementos de geometría racional. Tomo I: Geometría plana. Madrid, 1933. 296 pp.

ROBSON (A.). See DURELL (C. V.).

SULLIVAN (M. H.). The number and reality of the non-self-symmetric quadrilaterals in-and-circumscribed to the rational unicuspidal quartic with a line of symmetry. (Dissertation, Catholic University.) Washington, Catholic University of America, 1934. 17 pp.

THOMAS (T. Y.). The differential invariants of generalized spaces. Cambridge, University Press, 1934. 10+241 pp.

THOMPSON (A. J.). Logarithmetica Britannica. Being a standard table of logarithms to 20 decimal places. Part 1: Numbers 10,000 to 20,000. (Issued by the Biometric Laboratory, University of London.) London, Cambridge University Press, 1934. 4+100 pp.

ULLRICH (E.). See LINDELÖF (E.).

VERA (F.). Historia de la matemática en España. III: Arabes y Judios. Primera parte: Siglos VIII-XI. Madrid, Suarez, 1933. 310 pp.

——— Historia de la matemática en España. IV: Arabes y Judios. Segunda parte: Siglos XII–XVI. Madrid, Suarez, 1933. 285 pp.

VYGODSKY (M. I.). See ZEUTHEN (H. G.).

WEYL (H.). Mind and nature. Philadelphia, University of Pennsylvania Press, 1934. 100 pp.

WIND (E.). Das Experiment und die Metaphysik. Tübingen, Mohr, 1934. 12 +120 pp.

WINTER (F.). Das Spiel der 30 bunten Würfel. MacMahons Problem. Leipzig, Teubner, 1934.

ZEUTHEN (H. G.). History of mathematics in the sixteenth and seventeenth centuries. Translated from the German by P. Novikov. Revised with notes and preface by M. I. Vygodsky. Moscow and Leningrad, State Technical-Theoretical Publishing Company, 1933. (In Russian.) 429 pp.

PART II. APPLIED MATHEMATICS

ABDULLAH (C.). Notions de photogrammétrie terrestre et aérienne. Paris, Baillière, 1934.

AUDUBERT (R.). Phénomènes photoélectrochimiques, action de la lumière sur le potentiel métal-solution. (Actualités Scientifiques et Industrielles, No. 91: Réunion Internationale de Chimie-Physique, 1933, No. 11.) Paris, Hermann, 1934. 2+32 pp.

BENEDICKS (C.). L'effet électro-thermique homogène. Paris, Hermann, 1934. 30 pp.

BLOCH (F.). Les électrons dans les métaux; problèmes statiques, magnétisme. (Actualités Scientifiques et Industrielles, No. 86: Réunion Internationale de Chimie-Physique, 1933, No. 6.) Paris, Hermann, 1934. 2+20 pp.

BOHR (N.). Atomic theory and the description of nature. 1: Four essays, with an introductory survey. Cambridge, University Press, 1934. 6+119 pp.

BOLL (M.). See URBAIN (G.).

BRILLOUIN (L.). Les électrons dans les métaux du point de vue ondulatoire. (Actualités Scientifiques et Industrielles, No. 88: Réunion Internationale de Chimie-Physique, 1933, No. 8.) Paris, Hermann, 1934. 4+32 pp.

——— Conductibilité électrique et thermique des métaux. (Actualités Scientifiques et Industrielles, No. 89: Réunion Internationale de Chimie-Physique, 1933, No. 9.) Paris, Hermann, 1934. 3+71 pp.

BRONSTEIN (M. P.), DUKELSKI (W. M.), IWANENKO (D. D.), and KHARITON (U. W.), edited by. The atomic nucleus. (Problems of Modern Physics, No. 24.) Leningrad and Moscow, Izdatel'stvo, 1934. (In Russian.)

CAJORI (F.). See NEWTON (I.).

CROWTHER (J. G.). The progress of science. An account of recent fundamental researches in physics, chemistry and biology. London, Kegan Paul, 1934. 10+304 pp.

DARD (A.). L'onde, énergie gratuite. Paris, Etienne Chiron, 1934.

DUBOIS (E.). L'effet Volta. (Actualités Scientifiques et Industrielles, No. 81: Réunion Internationale de Chimie-Physique, 1933, No. 1.) Paris, Hermann, 1934. 5+21 pp.

DUKELSKI (W. M.). See BRONSTEIN (M. P.).

DURAND (G.). Sur les petits mouvements d'un système infini de tourbillons autour d'une position d'équilibre. (Publications Scientifiques du Ministère de l'Aire, No. 35.) Paris, Gauthier-Villars, 1933. 26 pp.

DURAND (W. F.), edited by. Aerodynamic theory. A general review of progress. (Under a grant of Guggenheim Fund for the Promotion of Aeronautics.) Volume 1, by W. F. Durand, M. M. Munk, R. Giacomelli, E. Pistolesi. Berlin, Springer, 1934. 15+398 pp.

DUTOIT (P.). Sur le potentiel métal-solution dans les dissolvants autres que l'eau. (Actualités Scientifiques et Industrielles, No. 93: Réunion Internationale de Chimie-Physique, 1933, No. 13.) Paris, Hermann, 1934. 13 pp.

EUCKEN (A.) und WOLF (K. L.) herausgegeben von. Hand- und Jahrbuch der chemischen Physik. Band 9: Die Spektren Entstehung und Zusammenhang mit der Struktur der Materie. Abschnitt 1: Atomspekten, von H. Kuhn. Leipzig, Akademische Verlagsgesellschaft, 1934. 6+266 pp.

FANTOZZI (L.). Les deux masses: hypothèses sur la force et la mécanique dans la nature. Paris, Marcel Rivière, 1934. 8+181 pp.

FAWDRY (R. C.). Examples in elementary statics and dynamics. Extracted from the first parts of "Statics" and "Dynamics." (Bell's Mathematical Series.) London, Bell, 1934. 7+145+16 pp.

GIACOMELLI (R.). See DURAND (W. F.).

HEYROVSKY (J.). A polarographic study of the electrokinetic phenomena of adsorption, electro-reduction and overpotential displayed at the dropping mercury cathode. (Actualités Scientifiques et Industrielles, No. 90: Réunion Internationale de Chimie-Physique, 1933, No. 10.) Paris, Hermann, 1934. 4+48 pp.

IWANENKO (D. D.). See BRONSTEIN (M. P.).

JANKOWSKI (K.). Asymétrie de la terre. Warszaw, the author, 1932. 154 pp.

JOFFÉ (A. F.). Conductibilité électrique des isolants solides et des semi-conducteurs. (Actualités Scientifiques et Industrielles, No. 87: Réunion Internationale de Chimie-Physique, 1933, No.*7.) Paris, Hermann, 1934. 6 +35 pp.

KAUFMANN (W.). Angewandte Hydromechanik. Band 2: Ausgewählte Kapitel aus der technischen Strömungslehre. Berlin, Springer, 1934. 7+293 pp.

KHARITON (U. W.). See BRONSTEIN (M. P.).

KUHN, (H). See EUCKEN (A).

KLAUTKE (F.). Spannen im Maschinenbau. Berlin, Springer, 1934. 54 pp.

LAGALLY (M.). Mechanik und Thermodynamik des stationären Gletschers. (Ergebnisse der Kosmischen Physik.) Leipzig, Akademische Verlagsgesellschaft, 1934. 90 pp.

LANGEVIN (P.). La notion de corpuscules et d'atomes. (Actualités Scientifiques et Industrielles, No. 132: Réunion Internationale de Chimie-Physique, 1933, No. 16.) Paris, Hermann, 1934.

LEHR (E.). Spannungsverteilung in Konstruktionselementen. Berlin, Verein Deutscher Ingenieure, 1934. 64 pp.

MASSA (F.). See OLSON (H. F.).

MOTTE (A.). See NEWTON (I.).

MUNK (M. M.). See DURAND (W. F.).

NEWTON (I.). Sir Isaac Newton's Mathematical principles of natural philosophy and his System of the world. Translated into English by Andrew Motte in 1729. The translations revised, and supplied with an historical and explanatory appendix, by Florian Cajori. Berkeley, University of California Press, 1934. 26+680 pp.

NORDHEIM (L.). Die Theorie der thermoelektrischen Effekte; Legierungen, unvollständige Ketten, Benedickseffekt. (Actualités Scientifiques et Industrielles, No. 131: Réunion Internationale de Chimie-Physique, 1933, No. 15.) Paris, Hermann, 1934. 3+23 pp.

OLSON (H. F.) and MASSA (F.). Applied acoustics. Philadelphia, Blakiston, 1934. 444 pp.

PISTOLESI (E.). See DURAND (W. F.).

PLANCK (M.). See ZIMMER (E.).

RIDEAL (E. K.). On phase boundary potentials. (Actualités Scientifiques et Industrielles, No. 83: Réunion Internationale de Chimie-Physique, 1933, No. 3.) Paris, Hermann, 1934. 17 pp.

SCARPA (O.). Pile metalliche che funzionano in eccezione alla legge delle tensioni elettriche nei circuiti metallici. (Actualités Scientifiques et Industrielles, No. 84: Réunion Internationale de Chimie-Physique, 1933, No. 4.) Paris, Hermann, 1934. 22 pp.

SPÄTH (W.). Theorie und Praxis der Schwingungsprüfmaschinen. Anleitung zur Ausführung und Auswertung dynamischer Untersuchungen mit Hilfe künstlicher Erschütterungen. Berlin, Springer, 1934. 6+98 pp.

STETSON (H. T.). Earth, radio and the stars. New York, McGraw-Hill, 1934. 336+18 pp.

STUART (H. A.). Molekulstruktur. Bestimmung von Molekulstrukturen mit physikalischen Methoden. Berlin, Springer, 1934. 388 pp.

TARDI (P.). Traité de géodésie. Fascicule 2: Astronomie géodésique de position, géodésie dynamique, la figure de la terre. Paris, Gauthier-Villars, 1934. Pp. 11+425–732.

THOMPSON (W. R.). Some notes on deflection. Application of area moment method to uniform section beams. London, Draughtsman Publishing Company, 1934. 27 pp.

TOLMAN (R. C.). Relativity thermodynamics and cosmology. Oxford, Clarendon Press, 1934. 502 pp.

URBAIN (G.) et BOLL (M.). La science, ses progrès, ses applications. Paris, Larousse, 1934.

VOLMER (M.). Das elektrolytische Kristallwachstum. (Actualités Scientifiques et Industrielles, No. 85: Réunion Internationale de Chimie-Physique, 1933, No. 5.) Paris, Hermann, 1934. 2+12 pp.

WARNER (F. M.). Applied descriptive geometry. With drafting-room problems. New York and London, McGraw-Hill, 1934. 226 pp.

WEGENER (K.). Die Physik der Erde. Eine Einführung in verständlicher Darstellung. Leipzig, Barth, 1934. 111 pp.

WHITEHEAD (T. N.). The design and use of instruments and accurate mechanism. Underlying principles. New York, Macmillan, 1934. 12+283 pp.

WILSON (H. A.). The mysteries of the atom. New York, Van Nostrand, 1934. 146+6 pp.

WILSON (M. A. H.). The electrical properties of semi-conductors and insulators: (Actualités Scientifiques et Industrielles, No. 82: Réunion Internationale de Chimie-Physique, 1933, No. 2.) Paris, Hermann, 1934. 14 pp.

WOLF (K. L.). See EUCKEN (A.).

ZIMMER (E.). Umsturz im Weltbild der Physik, gemeinverständlich dargestellt. Mit einem Geleitwort von Dr. Max Planck. München, Verlag Knorr und Hirth, 1934. 262 pp.

OFFICIAL COMMUNICATIONS

Meetings of the Society have been fixed at the following times and places:

NEW YORK CITY, October 27, 1934.

Abstracts must be in the hands of Associate Secretary J. R. Kline, 501 West 116th St., New York City, not later than September 29. Abstracts received by the Secretaries by September 7 will appear in the September issue of this Bulletin. By invitation of the Program Committee, Professor Emmy Noether, visiting lecturer at Bryn Mawr College, will deliver an address entitled *Modern hypercomplex series.*

LINCOLN, NEBRASKA, November 30-December 1, 1934.

Abstracts must be in the hands of Associate Secretary M. H. Ingraham, University of Wisconsin, Madison, Wisconsin, not later than November 3. All these will appear in the November issue of this Bulletin. By invitation of the Program Committee, addresses will be delivered by Professors G. A. Bliss and A. J. Kempner.

UNIVERSITY OF CALIFORNIA AT LOS ANGELES, December 1, 1934.

Abstracts must be in the hands of Associate Secretary T. M. Putnam, University of California, Berkeley, California, not later than November 3. All these will appear in the November issue of this Bulletin.

PITTSBURGH, PENNSYLVANIA, Annual Meeting, December 27-31, 1934.

Abstracts must be in the hands of Associate Secretary J. R. Kline, 501 West 116th St., New York City, not later than November 22, 1934. Abstracts received by the Secretaries by November 9 will appear in the November issue of this Bulletin.

R. G. D. RICHARDSON, *Secretary of the Society.*

NEW YORK CITY, February 23, 1935.

Abstracts will be due on or before January 26, 1935.

Articles for insertion in the BULLETIN should be addressed to E. R. HEDRICK, Editor of the BULLETIN, University of California at Los Angeles. Reviews should be sent to W. R. LONGLEY, Yale University, New Haven, Conn. Notes should be sent to H. W. KUHN, Ohio State University, Columbus, Ohio.

Subscriptions to the BULLETIN, orders for back numbers, and inquiries in regard to non-delivery of current numbers should be addressed to the American Mathematical Society, 450-459 Ahnaip St., Menasha, Wis., or 501 West 116th St., New York.

The initiation fees and the annual dues of members of the Society (see this BULLETIN, p. 322, May, 1930; and the List of Officers and Members, September, 1932, p. 66), are payable to the Treasurer of the Society, Professor G. W. Mullins, 501 West 116th St., New York City.

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